Fluctuating nonlinear hydrodynamics does not support an ergodic-nonergodic transition

Shankar P. Das[1] and Gene F. Mazenko[2]
[1]*School of Physical Sciences, Jawaharlal Nehru University, New Delhi 110067, India*
[2]*The James Franck Institute and Department of Physics, The University of Chicago, Chicago, Illinois 60637, USA*
(Received 11 January 2008; revised manuscript received 8 August 2008; published 12 February 2009)

Despite its appeal, real and simulated glass forming systems do not undergo an ergodic-nonergodic (ENE) transition. We reconsider whether the fluctuating nonlinear hydrodynamics (FNH) model for this system, introduced by us in 1986, supports an ENE transition. Using nonperturbative arguments, with no reference to the hydrodynamic regime, we show that the FNH model does not support an ENE transition. Our results support the findings in the original paper. Assertions in the literature questioning the validity of the original work are shown to be in error.

DOI: 10.1103/PhysRevE.79.021504 PACS number(s): 64.70.Q−

I. INTRODUCTION

It is appealing to associate the vitrification of a liquid into a frozen glassy state as an ergodic-nonergodic (ENE) transition. Unfortunately there is strong evidence against the ENE transition scenario in physical and numerical experiments. This is in agreement with the results we found 20 years ago in Ref. [1] (hereafter mentioned as DM) when we introduced the model of fluctuating nonlinear hydrodynamics (FNH). We present here a nonperturbative analysis of the FNH model and test the possibility of an ENE transition. In the end our results here agree with those in Ref. [1]. There is no sharp ENE transition in the FNH model. Recent reservations [2] concerning our results are shown to be unfounded.

In the theory of classical liquids, a new approach to studying the complex behavior of the supercooled state started with the introduction of the self-consistent mode-coupling theory (MCT) [4,5]. The model referred to here is based on a nonlinear feedback mechanism due to the coupling of the slowly decaying density fluctuations in the supercooled liquid. The feedback effects at metastable densities strongly enhance the transport properties of the liquid. In the simple version proposed initially [6–8] a sharp ergodic-to-nonergodic transition of the liquid into a glassy phase was predicted. This transition occurs at a critical density (or at the corresponding values of other controlling thermodynamic parameters) beyond which the density autocorrelation function freezes at a nonzero value over long times. Soon afterward it was demonstrated that this sharp ENE transition is [1] rounded. The absence of a sharp ENE transition in the supercooled liquids was supported by work [9,10] using similar theoretical models. Two recent works [2,3] have called these conclusions into question. We address these old questions here from a vantage point.

We organize this paper as follows. In the next section we briefly introduce the FNH model. This is followed by an analysis of whether this model supports an ENE transition. In Sec. III we compare our findings here to those in DM. Next we comment on the works which question the conclusions in DM. We end the paper with a short discussion.

II. FLUCTUATING NONLINEAR HYDRODYNAMIC MODEL

In Ref. [1] a model for the long time relaxation behavior of the supercooled liquid was constructed using fluctuating nonlinear hydrodynamics. The dynamics of collective modes in the liquid was formulated with nonlinear Langevin equations involving bare transport coefficients. These nonlinear stochastic equations for the time evolution of the conserved densities are plausible generalizations of the macroscopic hydrodynamic laws. The set of collective variables $\{\psi_i\}$ for the liquid we considered consists of mass and momentum densities $\{\rho(\mathbf{r},t),\mathbf{g}(\mathbf{r},t)\}$. The construction of the equations of motion [11] for the slow variables involve a driving free energy functional F which is expressed in terms of the hydrodynamic fields, i.e., ρ and $\mathbf{g}$. The corresponding equilibrium distribution for the system is $\exp(-\beta F)$. The free energy functional F is separated in two parts, $F=F_K[\mathbf{g},\rho]+F_U[\rho]$. The dependence of F on $\mathbf{g}$ is entirely in the kinetic part F_K in the form [12] constrained by Galilean invariance:

$$F_K[\mathbf{g},\rho]=\int d\mathbf{x}\frac{g^2(\mathbf{x})}{2\rho(\mathbf{x})}. \tag{1}$$

The potential part F_U is treated as a functional of the density only. The density ρ follows the continuity equation

$$\frac{\partial\rho}{\partial t}+\boldsymbol{\nabla}\cdot\mathbf{g}=0, \tag{2}$$

having the momentum density $\mathbf{g}$ as the flux which itself is a conserved property. The nonlinear equation for the momentum current density g_i is a generalized form of the Navier-Stokes equation [1],

$$\frac{\partial g_i}{\partial t}=-\sum_j\nabla_j\left[\frac{g_ig_j}{\rho}\right]-\rho\nabla_i\frac{\delta F_U}{\delta\rho}-\sum_j L^o_{ij}\frac{g_j}{\rho}+\theta_i. \tag{3}$$

The noise θ_i is assumed to be Gaussian following the fluctuation-dissipation relation with the bare damping matrix L^o_{ij}. For compressible liquids, the $1/\rho$ nonlinearity appears in two terms in the generalized Navier-Stokes equation. These are, respectively, the convective term coupling the flow fields and the dissipative term involving the bare viscosity of the liquid. The appearance of this nonlinearity in the hydrodynamic equations is formally avoided in Ref. [1] by introducing the local velocity field $\mathbf{V}(\mathbf{x},t)$,

$$\mathbf{g}(\mathbf{x},t) = \rho(\mathbf{x},t)\mathbf{V}(\mathbf{x},t). \tag{4}$$

The set of fluctuating variables in terms of which the renormalized field theory is constructed in our analysis therefore consists of the set $\psi_i \equiv \{\rho, \mathbf{g}, \mathbf{V}\}$. The consequences of the nonlinearities in the equations of motion, i.e., renormalization of bare transport coefficients, are obtained using graphical methods of field theory. The renormalized perturbation theory is developed in Ref. [1] using the standard approach of Martin-Siggia-Rose (MSR) field theory [13]. We follow the same here and for more details of this formalism we refer the reader to Ref. [1]. The correlation of the hydrodynamic fields involves averages defined in terms of the action $\mathcal{A}$ which is a functional of the field variables $\{\psi_i\}$ and the corresponding conjugate hatted fields $\{\hat{\psi}_i\}$ introduced in the MSR formalism. Using the equations of motions (2) and (3), respectively, for ρ and $\mathbf{g}$ the action functional is obtained as [1]

$$\mathcal{A} = \int dt \int d\mathbf{x}\Bigg[\sum_{i,j} \hat{g}_i \beta^{-1} L^o_{ij} \hat{g}_j + i\sum_i \hat{g}_i\left(\frac{\partial g_i}{\partial t} + \rho\nabla_i \frac{\delta F_u}{\delta\rho} + \sum_j \nabla_j(\rho V_i V_j) - \sum_j L^o_{ij} V_j\right) + i\hat{\rho}\left(\frac{\partial\rho}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{g}\right) + i\sum_i \hat{V}_i(g_i - \rho V_i)\Bigg]. \tag{5}$$

The theory is developed in terms of the correlation functions,

$$G_{\alpha\beta}(12) = \langle \psi_\beta(2)\psi_\alpha(1)\rangle, \tag{6}$$

and the response functions,

$$G_{\alpha\hat{\beta}}(12) = \langle \hat{\psi}_\beta(2)\psi_\alpha(1)\rangle. \tag{7}$$

The averages denoted here by the angular brackets are functional integrals over all the fields weighted by e^{-A}. The nonlinearities in the equations of motion (3) and (4) give rise to non-Gaussian terms in the action (5) involving products of three or more field variables. The role of the non-Gaussian parts of the action $\mathcal{A}$ on the correlation functions are quantified in terms of the self-energy matrix which show up in the equation satisfied by the response functions and that satisfied by the correlation functions. The self-energy matrix $\boldsymbol{\Sigma}$ is defined through the Dyson equation

$$\mathbf{G}^{-1} = \mathbf{G}_\mathbf{0}^{-1} - \boldsymbol{\Sigma}, \tag{8}$$

where $\mathbf{G_0}$ represents the Gaussian counterpart of $\mathbf{G}$ obtained by keeping only up to quadratic terms in the action $\mathcal{A}$. Let us first consider the cases in which both indices in the matrix equation (8) correspond to the unhatted fields. In this case,

(a) $[\mathbf{G_0^{-1}}]_{\alpha\beta}=0$ which follows from the action (5) obtained in the MSR field theory.

(b) $\boldsymbol{\Sigma}_{\alpha\beta}=0$ which follows from the causal nature of the response functions in the MSR field theory.

We therefore obtain that the elements of the $\mathbf{G}^{-1}$ matrix corresponding to the unhatted fields, $[\mathbf{G}^{-1}]_{\alpha\beta}=0$. Inverting the matrix $\mathbf{G}^{-1}$ having the above structure, we obtain for the correlation functions of the physical, unhatted field variables

TABLE I. The matrix of the coefficients $N_{\alpha\hat{\beta}}$ in the numerator on the right-hand side of Eq. (12) for the response functions.

	$\hat{\rho}$	$\hat{g}$	$\hat{V}$
ρ	$\omega\rho_L + iL$	$\rho_L q$	Lq
g	$q(\rho_L c^2 + L\gamma)$	$\rho_L\omega$	$L\omega$
V	$q(c^2 + i\omega\gamma)$	$\omega + iq^2\gamma$	$i(\omega^2 - q^2c^2)$

$$G_{\alpha\beta} = -\sum_{\mu\nu} G_{\alpha\hat{\mu}} C_{\hat{\mu}\hat{\nu}} G_{\hat{\nu}\beta}, \tag{9}$$

where Greek letter subscripts take values $\rho, \mathbf{g}, \mathbf{V}$, and the self-energy matrix $C_{\hat{\mu}\hat{\nu}}$ is given by

$$C_{\hat{\mu}\hat{\nu}} = 2\beta^{-1} L_0 \delta_{\hat{\mu}\hat{\nu}} \delta_{\hat{\mu},\hat{g}} - \Sigma_{\hat{\mu}\hat{\nu}}. \tag{10}$$

The double-hatted self-energies $\Sigma_{\hat{\mu}\hat{\nu}}$ vanish if either index corresponds to the density. Similarly from the set of equations denoted by (8) we obtain that the response functions satisfy

$$[(G_0^{-1})_{\hat{\alpha}\mu}(13) - \Sigma_{\hat{\alpha}\mu}(13)]G_{\mu\hat{\beta}}(32) = \delta(12)\delta_{\hat{\alpha}\hat{\beta}}. \tag{11}$$

The self-energies $\Sigma_{\hat{\alpha}\mu}$ are expressed in perturbation theory in terms of the two-point correlation and response functions. Using the explicit polynomial form of the action (5), the response functions are expressed in the general form,

$$G_{\alpha\hat{\mu}}(q,\omega) = \frac{N_{\alpha\hat{\mu}}(q,\omega)}{D(q,\omega)}, \tag{12}$$

where the matrix N is given in Table I and the determinant D in the denominator is given by

$$D(q,\omega) = \rho_L(\omega^2 - q^2c^2) + iL(q,\omega)(\omega + iq^2\gamma). \tag{13}$$

The various quantities are defined such that ρ_L, c^2, and L are identified as the corresponding renormalized quantities, respectively, for the bare density ρ_0, speed of sound squared c_0^2, and longitudinal viscosity L_0. We have in terms of single-hatted or response self-energies

$$\rho_L(q,\omega) = \rho_0 - i\Sigma_{\hat{V}V}(q,\omega), \tag{14}$$

$$L(q,\omega) = L_0 + i\Sigma_{\hat{g}V}(q,\omega), \tag{15}$$

$$qc^2(q,\omega) = qc_0^2 + \Sigma_{\hat{g}\rho}(q,\omega), \tag{16}$$

and γ is defined in terms of the self-energy element $\Sigma_{\hat{V}\rho} \equiv q\gamma$.

This model does not have a complete set of fluctuation-dissipation relations (FDR) linearly relating correlation and response functions. However, using the time translational invariance properties of the action (5), we obtained in DM the following fluctuation-dissipation relation between correlation and response functions involving the field g in the form

$$G_{V_i\alpha}(q,\omega) = -2\beta^{-1}\,\mathrm{Im}\,G_{\hat{g}_i\alpha}(q,\omega), \tag{17}$$

where α indicates any of the fields $\{\rho, g, V\}$.

III. ERGODIC-NONERGODIC TRANSITION AND FNH

Does this model have an ENE transition? To answer this question we first pose the conditions for such a transition. What we mean by a nonergodic phase is that $G_{\rho\rho}(t)$ is nonzero in the long time limit. This is equivalent to the corresponding one-sided Laplace transform $G_{\rho\rho}(z)\sim 1/z$ or the Fourier transform $G_{\rho\rho}(\omega)$ developing a δ-function peak at zero frequency. This will imply that generalized transport coefficient $L(\omega)$ also has a δ-function peak. This conforms to the physics of the viscosity blowing up as one enters the ideal glass phase. In the nonlinear fluctuating hydrodynamic formulation of the dynamics discussed here, renormalization of the viscosity is obtained from the self-energy $\Sigma_{\hat{g}\hat{g}}$ and the one-loop contribution involves the product of the density correlation functions. Indeed the integral relation between $L(\omega)$ and $G_{\rho\rho}(t)$ gives rise to the nonlinear feedback mechanism which forms the very basis of the self-consistent mode coupling approach to glass physics. To summarize, an ENE transition is characterized by a persistent time dependence of the density correlation function and this implies a diverging viscosity or equivalently that the self-energy $\Sigma_{\hat{g}\hat{g}}$ blows up at small frequencies,

$$\Sigma_{\hat{g}\hat{g}} = -A\,\delta(\omega) + \{R\,.\,T\}, \tag{18}$$

where $\{R.T\}$ represents terms which are regular in the $\omega\to 0$ limit. In writing the above expression we are not ignoring the wave-vector dependence but suppressing it to keep the notation simple. We expect that c and ρ_L remain well behaved in the supercooled state in the $\omega\to 0$ limit.

Is the above assumption compatible with the set of Dyson equations corresponding to the MSR action (5)? Setting Eq. (18) back into Eq. (9) we obtain a $\delta(\omega)$ peak in $G_{\rho\rho}$ as long as the response function $G_{\rho\hat{g}}$ is not zero in the $\omega\to 0$ limit. This result follows simply by setting both α and β equal to ρ in Eq. (9). It is straightforward to obtain that the singular contribution of $G_{\rho\rho}$ comes from $\Sigma_{\hat{g}\hat{g}}$ in the form

$$G_{\rho\rho} = G_{\rho\hat{g}}\Sigma_{\hat{g}\hat{g}}G_{\hat{g}\rho} + \{R,T\}. \tag{19}$$

It is therefore necessary for an ENE transition to occur that the response function $G_{\rho\hat{g}}$ not vanish as $\omega\to 0$. The response functions $G_{\psi\hat{\psi}}$ are calculated from (12) where $N_{\psi\hat{\psi}}$ are as given in Table I. The response function $G_{\rho\hat{g}}$ has the form

$$G_{\rho\hat{g}} = \frac{N_{\rho\hat{g}}}{D} = \frac{\rho_L q}{D}, \tag{20}$$

where D is given by Eq. (13). This requires that ρ_L goes to a nonzero value in the zero frequency limit and the determinant D not blow up as $\omega\to 0$. We assume, with no reason to expect otherwise, that the $\omega\to 0$ limits of ρ_L, γ, c^2, and L are nonzero. With these assumptions $D(\omega\to 0)$ is not infinite and hence $G_{\rho\hat{g}}\neq 0$ in the low-frequency limit.

From the same relation (9) it also follows that the correlation functions $G_{\rho V}$ and G_{VV} have a δ-function contribution due to $\Sigma_{\hat{g}\hat{g}}$ provided that $G_{\hat{g}V}$ is nonzero in the low-frequency limit. This is the case if the self-energy contribution $\gamma(\omega=0)\neq 0$. To demonstrate this we consider the case in which α and β in Eq. (9) are both equal to the current V to obtain

$$G^L_{VV} = G_{V\hat{g}}\Sigma_{\hat{g}\hat{g}}G_{\hat{g}V} + \{R,T\}. \tag{21}$$

Similarly $G_{\rho V}$ is obtained by setting α and β, respectively, equal to ρ and V,

$$G^L_{\rho V} = G_{\rho\hat{g}}\Sigma_{\hat{g}\hat{g}}G_{\hat{g}V} + \{R,T\}. \tag{22}$$

Using the expressions for $N_{\alpha\hat{\beta}}$ as provided in Table I, it follows, respectively, from Eqs. (21) and (22) that the correlation functions G_{VV} and $G_{\rho V}$ both have a diverging contribution in the $\omega\to 0$ limit if the quantity γ (and hence $\Sigma_{\hat{V}\rho}$) is nonvanishing in the same limit. To summarize, from Eq. (9) it thus follows that all three correlation functions $G_{\rho\rho}$, $G_{\rho V}$, and G_{VV} show a $\delta(\omega)$ component provided that γ is nonzero. On the other hand, the correlation functions involving a momentum index g do not show a δ-function peak at zero frequency. To demonstrate this we note that if either of α or β index in the left-hand side of (9) is the momentum density g then the singular contribution due to $\Sigma_{\hat{g}\hat{g}}$ is coupled to the response function $G_{g\hat{g}}$. However, from Table I it follows that

$$G_{g\hat{g}} = \frac{\rho_L\omega}{D(\omega)} \tag{23}$$

vanishes as $\omega\to 0$ as long as $D(\omega=0)\neq 0$. Therefore, the correlation functions involving a momentum index g therefore do not show a δ-function peak at zero frequency.

Next we consider the implications of the fluctuation-dissipation theorem (FDT) (17) on the above results. Since $G_{V\rho}$ and G_{VV} blow up, it then follows from the FDT that the imaginary parts of the response functions $G_{\hat{g}\rho}$ and $G_{\hat{g}V}$, respectively, blow up. Considering the explicit form of these response functions from Table I, we, respectively, obtain

$$G_{V\rho} = -2\beta^{-1}\frac{\mathrm{Im}(\rho_L D^*)}{DD^*}, \tag{24}$$

$$G_{VV} = -2\beta^{-1}\frac{\mathrm{Im}[\rho_L(\omega + iq^2\gamma)D^*]}{DD^*}. \tag{25}$$

This implies that we require simultaneously that D^*D is bounded, and imaginary parts of both $\rho_L qD^*$ and $(\omega + iq^2\gamma)D^*$ diverge. But since both D' and D'', respectively, denoting the real and imaginary parts of D are bounded, the quantities ρ_L and γ must diverge. However, if these latter quantities blow up then from (13) it follows that D must also blow up and we have a contradiction. The obvious conclusion is that the original assumption of a nonergodic phase is not supported in the model. The key self-energy contribution is γ. If for some reason this quantity vanishes at zero frequency then $G_{\rho V}$ and G_{VV} vanish as ω goes to zero. Then $G_{\rho V}$ and G_{VV} do not show a $\delta(\omega)$ component and one does not have the constraints on ρ_L, γ, and D. In this case one may have an ENE transition in this model. The presence of the nonzero self-energy $\Sigma_{\rho\hat{V}}$ is therefore crucial and ensures the absence of the ENE transition.

IV. RELATION TO DM RESULTS

The argument we give in the hydrodynamic regime in Ref. [1] is completely consistent with the results presented above. The simplest way of understanding the argument in the preceding section is to look at the response function

$$G_{\rho\hat{\rho}} = \frac{\omega\rho_L + iL}{\rho_L(\omega^2 - q^2c^2) + iL(\omega + iq^2\gamma)}. \tag{26}$$

The renormalization of the longitudinal viscosity L is computed, see Eq. (15) in terms of the longitudinal part $\Sigma^L_{\hat{g}V}$ of the corresponding self-energy matrix $\Sigma_{\hat{g}_i V_j}$ of the isotropic liquid,

$$L(q,\omega) = L_0 + \frac{\beta}{2}\Sigma^L_{\hat{g}V}(q,\omega). \tag{27}$$

If we ignore the self-energy $\Sigma_{\hat{V}\rho}$, the expression (26) is identical to the conventional expression for the density correlation function with the generalized memory function or the renormalized transport coefficient $L(q,\omega)$. The dependence of $G_{\rho\hat{\rho}}$ on the self-energy $\Sigma_{\hat{V}\rho}$ in the renormalized theory is a consequence of the nonlinear term involving the $\hat{V}$ field in the MSR action (5) and is originating from the nonlinear constraint (4) introduced to deal with the $1/\rho$ nonlinearity in the hydrodynamic equations. Analyzing the expression (9) for the correlation functions and the FDT relation (17) we obtain in the hydrodynamic limit the following nonperturbative relation between the two types of self-energies contributing alternatively to the renormalization of the longitudinal viscosity:

$$\gamma_{\hat{g}\hat{g}}(0,0) = 2\beta^{-1}\{\gamma'_{\hat{g}V}(0,0) + \lim_{\omega\to 0}[\gamma''_{\rho\hat{g}}(0,\omega)/\omega]\}, \tag{28}$$

where we have used in the above following definitions, in the isotropic limit, $\Sigma^L_{\hat{g}\hat{g}} \sim -q^2\gamma_{\hat{g}\hat{g}}$, $\Sigma^L_{\hat{g}V} \sim -iq^2\gamma_{\hat{g}V}$, and $\Sigma_{\rho\hat{g}} \sim q\gamma_{\rho\hat{g}}$. The relation (28), which is obtained from the FDT relation (17) only, implies that both the self-energies $\Sigma_{\hat{g}\hat{g}}$ and $\Sigma_{\hat{g}V}$ have the same diverging contribution in the low-frequency limit. In the simplified model it is this contribution in terms of the density correlation function which constitutes the feedback mechanism of MCT and leads to the dynamic transition beyond a critical density. The singular contribution to the renormalized transport coefficient L in (26) is now obtained in terms of the self-energy $\Sigma_{\hat{g}V}$. As a consequence of (28) it also follows that the response function $G_{\rho\hat{\rho}}$ is equal to the corresponding density correlation function $G_{\rho\rho}$ in the hydrodynamic limit. It is important to note here that (contrary to the assertion in Ref. [2]) this relation is not forced by us, rather it follows as a natural consequence of the relation (28) linking the response to correlation self-energies.

The asymptotic behavior of the density correlation function is inferred from $G_{\rho\hat{\rho}}$. The denominator of (26) for the response functions contain the self-energy $\Sigma_{\hat{V}\rho}$ which in this case is crucial for the long time dynamics and understanding how the ENE transition is cut off. The density correlation function (in the small q,ω limit) only freezes due to the feedback mechanism if the self-energy matrix element $\Sigma_{\hat{V}\rho}$ is zero—a result obtained in the earlier section. In this regard it is useful to note that for the $\omega\to 0$ limit the quantity $L(\omega + i\gamma q^2)$ in D does not diverge even when $L\sim 1/\omega$ is getting large, since $L\gamma q^2$ remains finite in the nonhydrodynamic regime $\omega\sim q^2$. The leading order in wave numbers $q\Sigma_{\hat{V}\rho}(q,0) \equiv q^2\gamma$ is expressed in terms of the self-energy $\Sigma^L_{\hat{V}\hat{V}}$ using the nonperturbative relation

$$\gamma_{\hat{V}\hat{V}}(0,0) = \frac{2\rho\beta^{-1}}{c^2}\gamma'_{\rho\hat{V}}(0,0), \tag{29}$$

where c is the sound speed introduced in (16). Note that the relation (29) is also obtained from the same fluctuation-dissipation relation (17). Though the absence of the sharp transition is proved in the general case as shown in the preceding section, the one-loop expression that we use for the cutoff function is a result that is valid only in the hydrodynamic limit.

V. RESPONSE TO CRITICISMS

We now address the criticisms of our work made recently in Ref. [2] and Ref. [3] (mentioned hereinafter, respectively, as ABL and CR). ABL imply that we misapplied the FDT relating $G_{\rho\rho}$ and $G_{\rho\hat{\rho}}$ in the hydrodynamical limit. These authors offer that we assumed a linear FDT from the beginning. The original paper of DM clearly discusses the consequences of not having a complete set of FD relations. In this work we only assumed its validity in the hydrodynamic limit to reach a closed equation for the density correlation function at the one-loop order. The absence of the sharp transition is proved nonperturbatively and it is not dependent on the existence of this FDT as is clearly demonstrated here. ABL in their paper have proposed a set of transformations which keeps the MSR action invariant. This gives rise to a set of linear fluctuation-dissipation theorems involving new fields. In the original DM work a subset of these FDT relations given by Eq. (17) was already obtained using time reversal symmetry. These FDT relations involve the field variable $(\delta F/\delta g_i) = g_i/\rho = V_i$. It should be noted here that the field V_i also appears in the equations of nonlinear fluctuating hydrodynamics. Indeed it is this FDT that proves to be most useful in our analysis. ABL add to this list another set of new FDTs through the introduction of this new field $\theta = (\delta F/\delta\rho)$. But how this newly found FDT leads to the conclusion that our analysis on the absence of transition being invalid remains puzzling. These authors miss the point that the argument on the absence of the ENE transition is not linked to a linear FDT between $G_{\rho\rho}$ and $G_{\rho\hat{\rho}}$. Indeed it is the other FDT [Eq. (17)] which is crucial in establishing the renormalizability of the dynamics in the hydrodynamic limit. It is also useful to note that the field θ involved in the new FDT proposed by ABL is actually absent in the nonlinear fluctuating hydrodynamic equations. The nonlinear part of this functional derivative of the free energy F with respect to density comes from the $(\delta F_K/\delta\rho)$ part. However, this term finally leads to the well-known Navier-Stokes nonlinearity $\nabla_j(g_i V_j)$ in the equations of motion and θ drops out from the dynamics.

In Ref. [3] the cutoff mechanism of Ref. [1] has been questioned by treating the highly nonlinear model described above using a quasilinear approach. CR basically make some phenomenological manipulations on a Newtonian dynamics model [14], ending with a memory function description that they claim, without proof, is related to our model. All subsequent discussions of our work made by these authors are based on this claim. Our model, as shown on examination of Table I, satisfies at all stages the density conservation law. The memory function proposed in CR to represent our work, Eq. (5) there, does not satisfy this conservation law. Therefore, the analysis of CR does not apply to the model we studied. None of their conclusions concerning our work are valid. CR concede that there is no error in our calculation; rather they claim that our model itself is the problem. The source of their error appears to be the assumption that our model can be represented in terms of a single memory function [15]. This work [3] represents a fundamental misunderstanding of the problem.

Interestingly, though the authors of both papers, ABL and CR, seem to agree that finally the ENE transition does not survive, they disagree with our analysis of the problem. The arguments put forward in Ref. [3] to rediscover that the transition is finally cut off are rather vague and of descriptive nature. These authors only seem to conjecture that the transition will be cut off nonperturbatively citing other recent works [16]. It is important to note here that the MSR field theoretic approach obtains the self-consistent MCT model in the most straightforward manner. We have further demonstrated here that this ENE transition does not survive from a nonperturbative analysis. However, the validity of the renormalized perturbation theory in terms of correlation functions has only been obtained in the hydrodynamic limit. At the one-loop order, the simplest form of this model obtains the standard MCT with a sharp dynamic transition. The explicit one-loop expression for the cutoff function of the ergodicity restoring mechanism has only been reached in the hydrodynamic limit. The extension of the equations of fluctuating nonlinear hydrodynamics to large wave vectors is only a plausible assumption at this point. In this regard we do believe that be it through the so-called memory function formalism or kinetic theory approaches, no one has been able to establish the renormalized theory for self-consistent MCT for all wave vectors. This indeed goes back to the very basic problem of extending the dense liquid state theory to the finite wave number and frequency limit and remains as a future challenge.

VI. DISCUSSION

The basic feedback mechanism of MCT is a consequence of simple quadratic nonlinearities in density fluctuations (arising from a purely dynamic origin) that is present in the pressure term of the generalized Navier-Stokes equation. Our work also establishes in a nonperturbative manner how the mode coupling model is obtained in the self-consistent form using the MSR field theory. When considered at the one-loop order this obtains the simple MCT model with the dynamic transition. The ergodicity restoring mechanism goes beyond this. The description in terms of coupling to currents is a physically appealing way of explaining the nature of the FNH equations (expressed in a form which can be sensibly related to the hydrodynamics of liquids). It is in fact the full implications of the density nonlinearities in the dynamics that cuts off the sharp transition to nonergodicity. This is also reflected in the fact that the basic conclusions of Ref. [1] follow even if the relevant nonlinearity is considered in a different manner. In fact by formulating the model [17] only in terms of the fields $\{\rho,\mathbf{g}\}$ the same conclusions implying the absence of the dynamic transition is reached as in Ref. [1]. The $1/\rho$ nonlinearity mentioned above is treated here in terms of a series of density nonlinearites. The self-energy matrix elements $\Sigma_{\hat{g}V}$ and $\Sigma_{\hat{V}\rho}$ are absent from the theoretical formulation in this case and the cutoff kernel is obtained here from a different self-energy element $\Sigma_{\hat{g}\rho}$.

Twenty years ago we had predicted that the feedback effects from mode coupling of density fluctuations, when properly analyzed keeping consistency with concepts of basic hydrodynamics, result in a qualitative crossover in the dynamics. We presented here a self-contained nonperturbative proof that FNH does not support an ENE transition. This analysis is completely compatible with the results of DM, simulations, and experiment.

ACKNOWLEDGMENT

S.P.D. acknowledges CSIR, India for financial support.

[1] S. P. Das and G. F. Mazenko, Phys. Rev. A **34**, 2265 (1986).

[2] A. Andreanov, G. Biroli, and A. Lefevre, J. Stat. Mech.: Theory Exp. (2006) P07008.

[3] M. E. Cates and S. R. Ramaswamy, Phys. Rev. Lett. **96**, 135701 (2006).

[4] S. P. Das, Rev. Mod. Phys. **76**, 785 (2004).

[5] B. Kim and G. F. Mazenko, Adv. Chem. Phys. **78**, 129 (1990).

[6] E. Leutheusser, Phys. Rev. A **29**, 2765 (1984).

[7] U. Bengtzelius, W. Götze, and A. Sjölander, J. Phys. C **17**, 5915 (1984).

[8] S. P. Das, G. F. Mazenko, S. Ramaswamy, and J. J. Toner, Phys. Rev. Lett. **54**, 118 (1985).

[9] R. Schmitz, J. W. Dufty, and P. De, Phys. Rev. Lett. **71**, 2066 (1993).

[10] W. Gőtze and L. Sjőjren, Z. Phys. B: Condens. Matter **65**, 415 (1987).

[11] S. K. Ma and G. F. Mazenko, Phys. Rev. B **11**, 4077 (1975).

[12] J. S. Langer and L. Turski, Phys. Rev. A **8**, 3230 (1973).

[13] P. C. Martin, E. D. Siggia, and H. A. Rose, Phys. Rev. A **8**, 423 (1973).

[14] E. Zaccarelli, G. Foffi, F. Sciortino, P. Tartaglia, and K. A. Dawson, Europhys. Lett. **55**, 157 (2001); E. Zaccarelli, P. De. Gregorio, G. Foffi, F. Sciortino, P. Tartaglia, and K. A. Dawson, J. Phys.: Condens. Matter **14**, 2413 (2002).

[15] K. Kawasaki, Physica A **208**, 35 (1994).

[16] P. Mayer, K. Miyazaki, and D. R. Reichman, Phys. Rev. Lett. **97**, 095702 (2006).

[17] G. F. Mazenko and J. Yeo, J. Stat. Phys. **74**, 1017 (1994).